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# QCD Amplitudes in the High-Energy Limit

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## Abstract

Dijet production with a rapidity gap between the jets is considered as a test ground for the production of a heavy Higgs boson via weak-boson fusion at hadron supercolliders. It is argued that in order to perform a detailed analysis of dijet production with a rapidity gap we need an  $\mathcal{O}(\alpha_s^4)$  calculation including the relevant collinear enhancements which give structure to the jets. Such a calculation needs not be exact, but must include the full leading power in  $\hat{s}/\hat{t}$  of the rate of dijet production in the high-energy limit. The QCD amplitudes must be determined to the corresponding accuracy. Accordingly, the scattering amplitudes necessary to compute the full leading power in  $\hat{s}/\hat{t}$  of dijet production to  $\mathcal{O}(\alpha_s^3)$  are analysed.

## 1 Rapidity gaps between jets

In recent years strong-interaction processes characterised by two large and disparate energy scales, which are typically the squared center-of-mass energy  $s$  and the squared momentum transfer  $t$ , with  $s \gg t$ , have been extensively analysed. The interest stems from the presumption that their description in terms of perturbative-QCD calculations at a

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fixed order in the coupling constant  $\alpha_s$  might not be adequate, and that a resummation at all orders of  $\alpha_s$  of large contributions of the type of  $\ln(s/t)$ , performed through the BFKL equation [1], might be needed.

These processes can be divided in two categories: *a*) inclusive processes, like deeply inelastic scattering (DIS) at small  $x_{bj}$ , dijet production in  $p\bar{p}$  collisions at large rapidity intervals, forward jet production in DIS; *b*) diffractive processes, like diffractive DIS, diffractive vector meson production, or dijet production in hadron collisions with a rapidity gap between the tagging jets.

The last of these processes, dijet production with a rapidity gap, is an example of double hard diffraction and is characterised by two large and disparate energy scales, the squared parton center-of-mass energy  $\hat{s}$  and a momentum-transfer scale of the order of the squared transverse energy of the jets  $E_{\perp}^2$ , and by a soft scale,  $\mu_s$ , the threshold energy for the detection of secondary hadrons within the rapidity gap. This process has been studied both at the Tevatron Collider [2] and in photoproduction at HERA [3]. Since the formation of a rapidity gap can happen only through the exchange of a colour singlet, which could be modeled by the BFKL resummation [4, 5], it is obvious in that respect the interest for dijet production with a rapidity gap.

However, the main motivation for the analysis of dijet production with a rapidity gap is to use it as a test ground for the production of a heavy Higgs boson at hadron supercolliders [6]. A Higgs boson is mainly produced via gluon fusion,  $gg \rightarrow H$ , mediated by a top-quark loop. The Higgs boson then decays mainly into a pair of  $W$  or  $Z$  bosons. Such a signal, though, is going to be swamped by the  $WW$  QCD and the  $t\bar{t}$  backgrounds. A heavy Higgs boson is also produced via weak-boson fusion,  $WW, ZZ \rightarrow H$ , though at a smaller rate [7], however such a production mechanism would have a distinctive radiation pattern with a gap in parton production in the central-rapidity region, because no color is exchanged between the quarks that emit the weak bosons [6, 8].

Producing a rapidity gap at the parton level is not sufficient though, since the gap is usually filled by soft hadrons produced in the rescattering between the spectators partons in the underlying event. Accordingly, Bjorken [6] introduced the gap survival probability,  $\langle |S^2| \rangle$ , i.e. the probability for a gap formed at the parton level to survive the rescattering between the spectators partons. The gap survival probability deals with the soft, i.e. low transverse-momentum, physics of the scattering between the two hadrons; therefore it can only be estimated using non-perturbative models [6, 9] (within perturbative QCD, the necessity of fulfilling the factorization theorems [10] would always allow for the emission of soft hadrons in the rescattering between the spectators partons [5]).

In addition, at the CERN LHC collider the requirement of running at very high luminosity will yield overlapping events in the same bunch crossing which are an additional source of soft hadrons and will further, and hopelessly, suppress the gap signal. A way out of this deadlock is to require a gap in minijet production [11, 12] rather than in soft-hadron production. This has the further advantage of dispensing with the gap survival probability, because the production of soft hadrons in the rescattering between the spectator partons is unrestricted, since the transverse energy of the minijets is of  $O(10 \text{ GeV})$ .

Modelling dijet production with a rapidity gap at the parton level is not easy: a rapidity gap in parton-parton scattering could be produced via  $\gamma$ ,  $W$ ,  $Z$ -boson exchange in the crossed channel; however, their rates turn out to be too small [13]. Single gluon exchange in the crossed channel, on the other hand, is likely not to produce a gap because the exchanged gluon being a color octet radiates off more gluons. Exchanging two gluons in the crossed channel in a color-singlet configuration is the simplest way of producing a gap at the parton level. However, this is an  $O(\alpha_s^4)$  process, for which no detailed calculation is available yet.

Bjorken [6] made an estimate of the order of magnitude of the rate for two-gluon exchange to be about 10% of the one-gluon exchange rate,  $\hat{\sigma}_{sing}/\hat{\sigma}_{oct} \sim 0.1$ . In the limit of  $\hat{s} \gg \hat{t}$ , Chehime and Zeppenfeld [14] made an  $O(\alpha_s^5)$  analysis of the bremsstrahlung pattern of a gluon emitted in parton-parton scattering with two-gluon singlet exchange; they found that if the transverse momentum  $p_{\perp rad}$  of the radiated gluon is of the same order as the transverse momenta  $p_{\perp jet}$  of the tagging jets then the gluon is radiated mainly in the central-rapidity region, like in the one-gluon exchange case; if  $p_{\perp rad} \ll p_{\perp jet}$ , then the gluon is radiated mainly in the forward direction; this is in agreement with the classical expectation that if the bremsstrahlung gluon is hard it has a short wavelength and may resolve the color structure of the two-gluon exchange; if it is soft its resolving power is low and sees the two exchanged gluons as a color singlet.

In addition, in the limit of  $\hat{s} \gg \hat{t}$ , the leading logarithmic (LL) contributions in  $\ln(\hat{s}/|\hat{t}|)$  to the radiative corrections to dijet production with a rapidity gap can be resummed at all orders of  $\alpha_s$  through the BFKL equation. Resumming the leading virtual radiative corrections to one-gluon exchange in parton-parton scattering, one obtains a Sudakov suppression [5]; namely, the production rate decreases exponentially as the rapidity gap width  $\Delta y \simeq \ln(\hat{s}/|\hat{t}|)$  increases. On the contrary, the resummation of the leading virtual

radiative corrections to two-gluon singlet exchange yields a production rate [4, 5]

$$\frac{d\hat{\sigma}_{sing}}{d\hat{t}} \simeq \frac{81\pi^3\alpha_s^4}{4\hat{t}^2} \frac{\exp[24\ln 2\alpha_s \Delta y/\pi]}{[21\zeta(3)\alpha_s\Delta y/2]^3}, \quad (1)$$

with  $\zeta(3) = 1.20206\dots$ , which is initially approximately flat, due to the competing effects of  $\Delta y$  in the exponential in the numerator and in the cube in the denominator, then it increases exponentially with  $\Delta y$ . This is in qualitative agreement with the data [2], which show that the rate of dijet production with a rapidity gap falls off exponentially at small gap widths (when the scattering is dominated by one-gluon exchange), and it flattens out at larger gap widths (when the scattering starts being dominated by two-gluon singlet exchange). However, no final rise of the gap production rate is observed.

The BFKL resummation, used in ref. [4, 5], as well the analysis of ref. [14] in the high-energy limit  $\hat{s} \gg \hat{t}$ , are not suitable for a detailed study of jet production because they are leading order analyses and their jets are leading partons. This drawback is even more acute for dijet production with a rapidity gap because the experiments measure the pseudorapidity gap width,  $\Delta\eta_c$ , between the edges of the jet cones, which differs from the pseudorapidity difference,  $\Delta\eta$ , between the jet centers by the cone sizes  $R$ ,  $\Delta\eta_c = \Delta\eta - 2R$ . The BFKL approximation is not able to distinguish between  $\Delta\eta$  and  $\Delta\eta_c$ .

In order to examine the gap fraction as a function of the gap width between the jet-cone edges, while accounting properly for the cone structures, we need a next-to-leading order calculation which includes, though, the basic features of color-singlet exchange. As we have mentioned, the simplest calculation of this kind for the dijet production rate with a rapidity gap is  $\mathcal{O}(\alpha_s^4)$ . At the moment an *exact* calculation of this kind is unfeasible because one needs to know two-loop parton-parton scattering amplitudes, which have not been computed yet. However, we want to argue that in order to analyse the basic features of color-singlet exchange it is not necessary to have an exact  $\mathcal{O}(\alpha_s^4)$  calculation; instead, it should be enough to know the full leading power in  $\hat{s}/\hat{t}$  of the rate of dijet production with a rapidity gap. This will contain all the collinear enhancements which give structure to the jets. Such calculation is not available yet either; however, albeit daunting, it should be much simpler than an exact  $\mathcal{O}(\alpha_s^4)$  calculation of dijet production.

In the next section we shall show that all the relevant collinear enhancements which appear in the exact  $\mathcal{O}(\alpha_s^3)$  calculation of dijet production are already present in the next-to-leading-logarithmic (NLL) corrections to the parton-parton scattering amplitudes.

## 2 QCD amplitudes in the high-energy limit

Let us consider the BFKL resummation [1], i.e. the LL approximation in  $\ln(\hat{s}/|\hat{t}|)$ , applied to dijet production; the QCD amplitudes we need in order to compute the BFKL resummation are of two kinds: *a*) virtual corrections to parton-parton scattering, in the high-energy limit  $\hat{s} \gg \hat{t}$  and in LL approximation; *b*) real corrections to parton-parton scattering, in the multi-Regge kinematics, i.e. in the strong rapidity ordering of the produced partons; when the scattering amplitudes are integrated over the phase space with multi-Regge kinematics, they yield the right power of  $\ln(\hat{s}/|\hat{t}|)$  to match the virtual corrections in LL approximation. The strong rapidity ordering constrains the scattering amplitudes not to have collinear singularities, because the squared invariant mass,  $m_{ij}^2$ , of any two produced partons is constrained to be large  $m_{ij}^2 \simeq |p_{i\perp}||p_{j\perp}|\exp(|y_i - y_j|)$ ; accordingly, the LL approximation lacks the running of the coupling constant, which must be considered fixed. Infrared singularities, though, are present in the amplitudes, both real and virtual, but they cancel after these are put together to compute the radiative corrections to dijet production.

### 2.1 QCD amplitudes in multi-Regge kinematics

A tree-level multigluon amplitude in a helicity basis can be written as a sum over color-ordered permutations of a color factor times a subamplitude, which is dependent on the helicities and momenta of the external partons [15]

$$M_n = \sum_{[a,1,\dots,n,b]'} \text{tr}(\lambda^a \lambda^{d_1} \dots \lambda^{d_n} \lambda^b) m(-p_a, -\nu_a; p_1, \nu_1; \dots; p_n, \nu_n; -p_b, -\nu_b), \quad (2)$$

where  $a, d_1, \dots, d_n, b$ , and  $\nu_a, \nu_1, \dots, \nu_b$  are respectively the colors and the helicities of the gluons, the  $\lambda$ 's are the color matrices in the fundamental representation of  $\text{SU}(N_c)$ , the sum is over the noncyclic permutations of the color orderings  $[a, 1, \dots, b]$  and all the momenta are taken as outgoing. In particular, for the *maximally helicity-violating* configurations  $(-, -, +, \dots, +)$ , the subamplitudes  $m(-p_a, -\nu_a; p_1, \nu_1; \dots; p_n, \nu_n; -p_b, -\nu_b)$ , assume the form [16],

$$m(-, -, +, \dots, +) = 2^{1+n/2} g^n \frac{\langle p_i p_j \rangle^4}{\langle p_a p_1 \rangle \dots \langle p_n p_b \rangle \langle p_b p_a \rangle}, \quad (3)$$

with  $i$  and  $j$  the gluons of negative helicity.

Before considering multiparton amplitudes in multi-Regge kinematics, we need to recall how a parton-parton scattering amplitude factorises in the high-energy limit. We

consider the elastic scattering of two gluons of momenta  $p_a$  and  $p_b$  in two gluons of momenta  $p_{a'}$  and  $p_{b'}$ , in the limit  $\hat{s} \gg |\hat{t}|$ . The corresponding amplitude is (cf. ref. [17] and the Appendix),

$$M_{\nu_a \nu_{a'} \nu_{b'} \nu_b}^{aa'bb'} = 2\hat{s} \left[ ig f^{aa'c} C_{-\nu_a \nu_{a'}}^{gg}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}} \left[ ig f^{bb'c} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \quad (4)$$

with  $p_{b\perp} = -p_{a\perp} = q_\perp$  and  $\hat{t} \simeq -|q_\perp|^2$ , and the helicity-conserving vertices  $g^* g \rightarrow g$ , with  $g^*$  an off-shell gluon,

$$C_{-+}^{gg}(-p_a, p_{a'}) = 1 \quad C_{-+}^{gg}(-p_b, p_{b'}) = \frac{p_{b'\perp}^*}{p_{b'\perp}}. \quad (5)$$

The  $C$ -vertices transform into their complex conjugates under helicity reversal,  $C_{\{\nu\}}^*(\{k\}) = C_{\{-\nu\}}(\{k\})$ . The helicity-flip vertex  $C_{++}$  is subleading in the high-energy limit.

The quark-gluon  $q g \rightarrow q g$  scattering amplitude in the high-energy limit is,

$$M^{qg \rightarrow qg} = 2\hat{s} \left[ g \lambda_{a'\bar{a}}^c C_{-\nu_a \nu_a}^{\bar{q}q}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}} \left[ ig f^{bb'c} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \quad (6)$$

$$M^{gq \rightarrow gq} = 2\hat{s} \left[ ig f^{aa'c} C_{-\nu_a \nu_{a'}}^{gg}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}} \left[ g \lambda_{b'\bar{b}}^c C_{-\nu_b \nu_b}^{\bar{q}q}(-p_b, p_{b'}) \right], \quad (7)$$

where we have labelled the incoming quarks as outgoing antiquarks by convention, and the antiquark is  $-p_a$  in eq.(6) and  $-p_b$  in eq.(7), and the  $C$ -vertices  $g^* q \rightarrow q$  are,

$$C_{-+}^{\bar{q}q}(-p_a, p_{a'}) = 1; \quad C_{-+}^{\bar{q}q}(-p_b, p_{b'}) = \left( \frac{p_{b'\perp}^*}{p_{b'\perp}} \right)^{1/2}. \quad (8)$$

The antiquark-gluon  $\bar{q} g \rightarrow \bar{q} g$  amplitude is found accordingly [17]. The quark-quark  $qq \rightarrow qq$  scattering amplitude in the high-energy limit is

$$M^{qq \rightarrow qq} = 2s \left[ g \lambda_{a'\bar{a}}^c C_{-\nu_a \nu_a}^{\bar{q}q}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}} \left[ g \lambda_{b'\bar{b}}^c C_{-\nu_b \nu_b}^{\bar{q}q}(-p_b, p_{b'}) \right]. \quad (9)$$

The amplitudes (4), (6), (7), (9), have all the effective form of a gluon exchange in the  $t$  channel, and differ only for the relative color strength in the production vertices [18]. This allows us to replace an incoming gluon with a quark, for instance on the upper line, via the simple substitution

$$ig f^{aa'c} C_{-\nu_a \nu_{a'}}^{gg}(-p_a, p_{a'}) \leftrightarrow g \lambda_{a'\bar{a}}^c C_{-\nu_a \nu_a}^{\bar{q}q}(-p_a, p_{a'}), \quad (10)$$

and similar ones for an antiquark and/or for the lower line.

Next, we consider the production of three gluons of momenta  $p_{a'}$ ,  $k$  and  $p_{b'}$ , and we require that the gluons are strongly ordered in their rapidities and have comparable transverse momenta,

$$y_{a'} \gg y \gg y_{b'}; \quad |p_{a'\perp}| \simeq |k_\perp| \simeq |p_{b'\perp}|. \quad (11)$$

Eq.(11) is the simplest example of *multi-Regge kinematics* (54). The amplitude is,

$$\begin{aligned} & M^{gg \rightarrow ggg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; k, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ &= 2\hat{s} \left[ ig f^{aa'c} C_{-\nu_a \nu_{a'}}^{gg}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}_1} \\ &\times \left[ ig f^{cdc'} C_\nu^g(q_1, q_2) \right] \frac{1}{\hat{t}_2} \left[ ig f^{bb'c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \end{aligned} \quad (12)$$

with  $p_{a'\perp} = -q_{1\perp}$ ,  $p_{b'\perp} = q_{2\perp}$  and  $\hat{t}_i \simeq -|q_{i\perp}|^2$  with  $i = 1, 2$  and with Lipatov vertex  $g^* g^* \rightarrow g$  [19, 20],

$$C_+^g(q_1, q_2) = \sqrt{2} \frac{q_{1\perp}^* q_{2\perp}}{k_\perp}. \quad (13)$$

The amplitude (12) has the effective form of a gluon-ladder exchange in the  $t$  channel. Again, we may replace an incoming gluon with a quark via the substitution (10). As we shall see in sect. 2.2, no quarks may be produced along the ladder since that would involve quark exchange in the  $t$  channel, which is suppressed in the kinematics (11). Eq.(12) generalizes to the production of  $n$  gluons in multi-Regge kinematics (54) in a straightforward manner [1, 20].

In the soft limit,  $k \rightarrow 0$ , a generic subamplitude in eq. (2) factorises as [15, 21],

$$\begin{aligned} & \lim_{k \rightarrow 0} m^{gg \rightarrow ggg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; k, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ &= m^{gg \rightarrow gg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \times \text{soft}(p_{a'}; k, \nu; p_{b'}), \end{aligned} \quad (14)$$

with eikonal factor [15, 22],

$$\text{soft}(p_{a'}; k, +; p_{b'}) = \sqrt{2} \frac{\langle p_{a'} p_{b'} \rangle}{\langle p_{a'} k \rangle \langle k p_{b'} \rangle}, \quad (15)$$

with spinor products (52), and  $\text{soft}(p_{a'}; k, -; p_{b'})$  obtained from  $\text{soft}(p_{a'}; k, +; p_{b'})$  by exchanging the  $\langle jk \rangle$  spinor products in eq. (15) with  $[kj]$  products. The eikonal factor (15) does not depend on the helicities of gluons  $a'$  and  $b'$ . Note that the soft factorization of eq. (14) does not carry on the full amplitude (2). Indeed, for the helicity configurations

allowed by eq. (14), eq. (12) reduces to

$$\begin{aligned}
& \lim_{k \rightarrow 0} M^{gg \rightarrow ggg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; k, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\
&= 2\hat{s} \left[ ig f^{aa'c} C_{-\nu_a \nu_{a'}}^{gg}(-p_a, p_{a'}) \right] \frac{1}{\hat{t}} \\
&\times \left[ ig f^{cdc'} \text{soft}(p_{a'}; k, \nu; p_{b'}) \right] \left[ ig f^{bb'c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right],
\end{aligned} \tag{16}$$

with

$$\text{soft}(p_{a'}; k, +; p_{b'}) = -\frac{\sqrt{2}}{k_\perp}, \tag{17}$$

and  $\text{soft}(p_{a'}; k, -; p_{b'}) = \text{soft}(p_{a'}; k, +; p_{b'})^*$ .

The square of the amplitude (12), integrated over the phase space of the intermediate gluon in multi-Regge kinematics (11) yields an  $O(\alpha_s \ln(s/|t|))$  correction to gluon-gluon scattering. Because of eq. (14), the real correction is infrared divergent. To complete the  $O(\alpha_s)$  corrections, and to cancel the infrared divergence, one needs the 1-loop gluon-gluon amplitude in LL approximation. The virtual radiative corrections to eq. (4) in LL approximation are obtained, to all orders in  $\alpha_s$ , by replacing [1],

$$\frac{1}{t} \rightarrow \frac{1}{t} \left( \frac{s}{-t} \right)^{\alpha(t)}, \tag{18}$$

in eq. (4), with  $\alpha(t)$  related to the loop transverse-momentum integration

$$\alpha(t) = \alpha_s N_c \hat{t} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 (q-k)_\perp^2}, \tag{19}$$

and  $\alpha_s = g^2/4\pi$ . The infrared divergence in eq. (19) can be regulated in 4 dimensions with an infrared-cutoff mass. Alternatively, using dimensional regularization in  $d = 4 - 2\epsilon$  dimensions, the integral in eq. (19) is performed in  $2 - 2\epsilon$  dimensions, yielding

$$\alpha(t) = 2g^2 N_c \frac{1}{\epsilon} \left( \frac{\mu^2}{-t} \right)^\epsilon c_\Gamma, \tag{20}$$

with

$$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon) \Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}. \tag{21}$$

Adding the 1-loop gluon-gluon amplitude, multiplied by its tree-level counterpart, to the square of the amplitude (12), integrated over the phase space of the intermediate gluon, cancels the infrared divergences and yields a finite  $O(\alpha_s \ln(s/|t|))$  correction to gluon-gluon scattering.



## 2.2 Real NLL corrections to QCD amplitudes

In order to compute the NLL corrections to the BFKL resummation [23], one needs: *a*) to compute the virtual corrections to NLL accuracy; *b*) to relax the strong rapidity ordering in the real corrections, by allowing any two partons to be produced with comparable rapidities. I shall illustrate how this comes about in a fixed-order calculation, by considering the NLL  $\mathcal{O}(\alpha_s)$  corrections to parton-parton scattering. Let three partons be produced with momenta  $k_1$ ,  $k_2$  and  $p_{b'}$  in the scattering between two partons of momenta  $p_a$  and  $p_b$ , and to be specific, I shall take partons  $k_1$  and  $k_2$  in the forward-rapidity region of parton  $p_a$ , the analysis for  $k_1$  and  $k_2$  in the forward-rapidity region of  $p_b$  being analogous,

$$y_1 \simeq y_2 \gg y_{b'}; \quad |k_{1\perp}| \simeq |k_{2\perp}| \simeq |p_{b'\perp}|. \quad (22)$$

First we consider the amplitude for the scattering  $g g \rightarrow g g g$ , [24, 25]

$$\begin{aligned} & M^{gg}(-p_a, -\nu_a; k_1, \nu_1; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ &= 2\hat{s} \left\{ C_{-\nu_a \nu_1 \nu_2}^{ggg}(-p_a, k_1, k_2) \left[ (ig)^2 f^{ad_1 c} f^{cd_2 c'} A_{\Sigma \nu_i}(-p_a, k_1, k_2, q) + \left( \begin{matrix} k_1 \leftrightarrow k_2 \\ d_1 \leftrightarrow d_2 \end{matrix} \right) \right] \right\} \\ & \times \frac{1}{\hat{t}} \left[ ig f^{bb'c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \end{aligned} \quad (23)$$

with  $C^{gg}$  as in eq. (8), and the production vertex  $g^* g \rightarrow g g$  of gluons  $k_1$  and  $k_2$  enclosed in curly brackets, and with  $\Sigma \nu_i = -\nu_a + \nu_1 + \nu_2$  and

$$\begin{aligned} C_{-++}^{ggg}(-p_a, k_1, k_2) &= 1; \quad C_{+-+}^{ggg}(-p_a, k_1, k_2) = \frac{1}{\left(1 + \frac{k_2^+}{k_1^+}\right)^2}; \\ C_{++-}^{ggg}(-p_a, k_1, k_2) &= \frac{1}{\left(1 + \frac{k_1^+}{k_2^+}\right)^2}; \quad A_+(-p_a, k_1, k_2, q) = -\sqrt{2} \frac{q_\perp}{k_{1\perp}} \sqrt{\frac{k_1^+}{k_2^+}} \frac{1}{\langle k_1 k_2 \rangle}; \end{aligned} \quad (24)$$

where the spinor product  $\langle k_1 k_2 \rangle$  is defined in eq.(52), and the momentum exchanged in the crossed channel is  $q = p_{b'} - p_b$ . The vertex  $C_{+++}^{ggg}(-p_a, k_1, k_2)$  is subleading in the high-energy limit.

In the multi-Regge limit  $k_1^+ \gg k_2^+$  the production vertex  $g^* g \rightarrow g g$  becomes

$$\lim_{k_1^+ \gg k_2^+} C_{-\nu_a \nu_1 \nu_2}^{ggg}(-p_a, k_1, k_2) A_{\Sigma \nu_i}(-p_a, k_1, k_2, q) = C_{-\nu_a \nu_1}^{gg}(-p_a, k_1) \frac{1}{\hat{t}_1} C_{\nu_2}^g(q_1, q), \quad (25)$$

with  $q_1 = p_a - k_1$ , and  $\hat{t}_1 \simeq -|q_{1\perp}|^2$ , thus the amplitude (23) reduces to the amplitude in multi-Regge kinematics (12), as expected.

The collinear factorization for a generic amplitude occurs both on the subamplitude and on the full amplitude [15], since in eq. (2) color orderings where the collinear gluons are not adjacent do not have a pole. Hence in the collinear limit for gluons  $i$  and  $j$ , with  $k_i = zP$  and  $k_j = (1 - z)P$ , a generic amplitude (2) can be written as

$$\lim_{p_i || p_j} M^{\dots d_i d_j \dots}(\dots; p_i, \nu_i; p_j, \nu_j; \dots) = M^{\dots c \dots}(\dots; P, \nu; \dots) \text{Split}_{-\nu}(p_i, \nu_i; p_j, \nu_j). \quad (26)$$

Accordingly, in the collinear limit,  $k_1 = zP$  and  $k_2 = (1 - z)P$ , using the Jacobi identity for the group structure functions we can write the amplitude (23) as

$$\begin{aligned} & \lim_{k_1 || k_2} M^{gg \rightarrow ggg}(-p_a, -\nu_a; k_1, \nu_1; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ &= M^{gg \rightarrow gg}(-p_a, -\nu_a; P, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \times \text{Split}_{-\nu}^{g \rightarrow gg}(k_1, \nu_1; k_2, \nu_2), \end{aligned} \quad (27)$$

with  $M^{gg \rightarrow gg}$  given in eq. (4), and with collinear factor

$$\text{Split}_{-\nu}^{g \rightarrow gg}(k_1, \nu_1; k_2, \nu_2) = ig f^{cd_1 d_2} \text{split}_{-\nu}^{g \rightarrow gg}(k_1, \nu_1; k_2, \nu_2), \quad (28)$$

with splitting factors [15],

$$\begin{aligned} \text{split}_{-}^{g \rightarrow gg}(k_1, +; k_2, +) &= \sqrt{2} \frac{1}{\sqrt{z(1-z)} \langle k_1 k_2 \rangle} \\ \text{split}_{+}^{g \rightarrow gg}(k_1, -; k_2, +) &= \sqrt{2} \frac{z^2}{\sqrt{z(1-z)} \langle k_1 k_2 \rangle} \\ \text{split}_{+}^{g \rightarrow gg}(k_1, +; k_2, -) &= \sqrt{2} \frac{(1-z)^2}{\sqrt{z(1-z)} \langle k_1 k_2 \rangle}, \end{aligned} \quad (29)$$

and  $\text{split}_{\nu}^{g \rightarrow gg}(k_1, -\nu_1; k_2, -\nu_2)$  obtained from  $\text{split}_{-\nu}^{g \rightarrow gg}(k_1, \nu_1; k_2, \nu_2)$  by exchanging  $\langle k_1 k_2 \rangle$  with  $[k_2 k_1]$ .

In the soft limit,  $k_1 \rightarrow 0$ , using the factorization of the subamplitude (14), eq. (23) reduces for the non-null subamplitudes to

$$\begin{aligned} & \lim_{k_1 \rightarrow 0} M^{gg \rightarrow ggg}(-p_a, -\nu_a; k_1, \nu_1; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) = 2\hat{s}(ig)^2 \\ & \times \left[ f^{ad_1 c} f^{cd_2 c'} \text{soft}(-p_a; k_1, \nu_1; k_2) + f^{ad_2 c} f^{cd_1 c'} \text{soft}(k_2; k_1, \nu_1; p_{b'}) \right] \\ & \times \frac{1}{\hat{t}} \left[ ig f^{bb' c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \end{aligned} \quad (30)$$

with eikonal factors,

$$\begin{aligned}\text{soft}(-p_a; k_1, +; k_2) &= \sqrt{2} \frac{\langle p_a k_2 \rangle}{\langle p_a k_1 \rangle \langle k_1 k_2 \rangle} = \sqrt{2} \frac{k_{2\perp}}{k_{1\perp}} \sqrt{\frac{k_1^+}{k_2^+}} \frac{1}{\langle k_1 k_2 \rangle} \\ \text{soft}(k_2; k_1, +; p_{b'}) &= \sqrt{2} \frac{\langle k_2 p_{b'} \rangle}{\langle k_2 k_1 \rangle \langle k_1 p_{b'} \rangle} = \sqrt{2} \sqrt{\frac{k_2^+}{k_1^+}} \frac{1}{\langle k_2 k_1 \rangle},\end{aligned}\quad (31)$$

and the eikonal factors for  $(k_1, -)$  obtained from eq. (31) by complex conjugation.

The amplitude for the production of a  $q\bar{q}$  pair in the forward-rapidity region of gluon  $p_a$  is [26],

$$\begin{aligned}M^{g g \rightarrow \bar{q} q g}(-p_a, -\nu_a; k_1, \nu_1; k_2, -\nu_1; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ = 2\hat{s} \left\{ g^2 C_{-\nu_a \nu_1 -\nu_1}^{g \bar{q} q}(-p_a, k_1, k_2) \left[ (\lambda^{c'} \lambda^a)_{d_2 \bar{d}_1} A_{-\nu_a}(k_1, k_2) + (\lambda^a \lambda^{c'})_{d_2 \bar{d}_1} A_{-\nu_a}(k_2, k_1) \right] \right\} \\ \times \frac{1}{t} \left[ i g f^{bb'c'} C_{-\nu_b \nu_{b'}}^{g g}(-p_b, p_{b'}) \right],\end{aligned}\quad (32)$$

with  $k_1$  the antiquark, the production vertex  $g^* g \rightarrow \bar{q} q$  in curly brackets,  $A$  defined in eq.(24), and  $C^{g \bar{q} q}$  given by,

$$\begin{aligned}C_{++-}^{g \bar{q} q}(-p_a, k_1, k_2) &= \sqrt{\frac{k_1^+}{k_2^+}} \frac{1}{\left(1 + \frac{k_1^+}{k_2^+}\right)^2} \\ C_{+-+}^{g \bar{q} q}(-p_a, k_1, k_2) &= \sqrt{\frac{k_2^+}{k_1^+}} \frac{1}{\left(1 + \frac{k_2^+}{k_1^+}\right)^2}.\end{aligned}\quad (33)$$

In the multi-Regge limit  $k_1^+ \gg k_2^+$  the vertex  $g^* g \rightarrow \bar{q} q$  in eq. (32) vanishes, i.e. as anticipated in sect. 2.1 quark production along the ladder is suppressed in the multi-Regge kinematics.

Like the purely gluonic amplitude (2), also a generic amplitude with a  $\bar{q} q$  pair factorises in the collinear limit according to eq. (26). Accordingly, for  $k_1 = zP$  and  $k_2 = (1-z)P$ , eq. (32) reduces to

$$\begin{aligned}\lim_{k_1 || k_2} M^{g g \rightarrow \bar{q} q g}(-p_a, -\nu_a; k_1, \nu_1; k_2, -\nu_1; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ = M^{g g \rightarrow g g}(-p_a, -\nu_a; P, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \times \text{Split}_{-\nu}^{g \rightarrow \bar{q} q}(k_1, \nu_1; k_2, -\nu_1),\end{aligned}\quad (34)$$

with  $M^{g g \rightarrow g g}$  given in eq. (4), and with collinear factor

$$\text{Split}_{-\nu}^{g \rightarrow \bar{q} q}(k_1, \nu_1; k_2, -\nu_1) = g (\lambda^c)_{d_2 \bar{d}_1} \text{split}_{-\nu}^{g \rightarrow \bar{q} q}(k_1, \nu_1; k_2, -\nu_1). \quad (35)$$

with splitting factors [15],

$$\begin{aligned}\text{split}_+^{g \rightarrow \bar{q}q}(k_1, +; k_2, -) &= \sqrt{2} \frac{z^{1/2}(1-z)^{3/2}}{\sqrt{z(1-z)}\langle k_1 k_2 \rangle} \\ \text{split}_+^{g \rightarrow \bar{q}q}(k_1, -; k_2, +) &= \sqrt{2} \frac{z^{3/2}(1-z)^{1/2}}{\sqrt{z(1-z)}\langle k_1 k_2 \rangle},\end{aligned}\quad (36)$$

and with  $\text{split}_\nu^{g \rightarrow \bar{q}q}(k_1, -\nu_1; k_2, \nu_1)$  obtained from  $\text{split}_{-\nu}^{g \rightarrow \bar{q}q}(k_1, \nu_1; k_2, -\nu_1)$  by exchanging  $\langle k_1 k_2 \rangle$  with  $[k_2 k_1]$ .

In the soft limits,  $k_1 \rightarrow 0$  or  $k_2 \rightarrow 0$ , the vertex  $g^* g \rightarrow \bar{q}q$  in eq. (32) yields at most a square-root singularity, whose square once integrated over the phase space does not yield an infrared divergence, in accordance to the general feature that there are no infrared divergences associated to soft fermions.

The amplitude for the production of a  $qg$  pair in the forward-rapidity region of quark  $p_a$  is

$$\begin{aligned}M^{qg \rightarrow qgg}(-p_a, -\nu; k_1, \nu; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ = 2\hat{s} \left\{ g^2 C_{-\nu\nu\nu_2}^{\bar{q}qg}(-p_a, k_1, k_2) \left[ \left( \lambda^{d_2} \lambda^{c'} \right)_{d_1 \bar{a}} A_{\nu_2}(k_1, k_2) - \left( \lambda^{c'} \lambda^{d_2} \right)_{d_1 \bar{a}} B_{\nu_2}(k_1, k_2) \right] \right\} \\ \times \frac{1}{\hat{t}} \left[ ig f^{bb'c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right],\end{aligned}\quad (37)$$

with  $k_1$  the final-state quark, and the production vertex  $qg^* \rightarrow qg$  in curly brackets, with  $A$  defined in eq.(24), and  $B$  given by

$$B_{\Sigma\nu_i}(-p_a, k_1, k_2) = A_{\Sigma\nu_i}(-p_a, k_1, k_2) + A_{\Sigma\nu_i}(-p_a, k_2, k_1), \quad (38)$$

and

$$\begin{aligned}C_{-++}^{\bar{q}qg}(-p_a, k_1, k_2) &= \frac{1}{\left(1 + \frac{k_2^+}{k_1^+}\right)^{1/2}}, \\ C_{+-+}^{\bar{q}qg}(-p_a, k_1, k_2) &= \frac{1}{\left(1 + \frac{k_2^+}{k_1^+}\right)^{3/2}}.\end{aligned}\quad (39)$$

In the multi-Regge limit  $k_1^+ \gg k_2^+$  the amplitude (37) reduces to eq.(12), with the substitution (10) for the upper line, and the vertex  $C^{\bar{q}q}$  in eq.(8).

In the collinear limit,  $k_1 = zP$  and  $k_2 = (1-z)P$ , the coefficient  $B$ , eq. (38), vanishes and eq. (37) reduces to

$$\begin{aligned} & \lim_{k_1 \parallel k_2} M^{qg \rightarrow qgg}(-p_a, -\nu; k_1, \nu; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ &= M^{qg \rightarrow qgg}(-p_a, -\nu; P, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \times \text{Split}_{-\nu}^{q \rightarrow qg}(k_1, \nu; k_2, \nu_2), \end{aligned} \quad (40)$$

with  $M^{qg \rightarrow qgg}$  given in eq. (6), and with collinear factor

$$\text{Split}_{-\nu}^{q \rightarrow qg}(k_1, \nu; k_2, \nu_2) = g(\lambda^{d_2})_{d_1 \bar{c}} \text{split}_{-\nu}^{q \rightarrow qg}(k_1, \nu; k_2, \nu_2), \quad (41)$$

with splitting factors [15],

$$\begin{aligned} \text{split}_{-}^{q \rightarrow qg}(k_1, +; k_2, +) &= \sqrt{2} \frac{z^{1/2}}{\sqrt{z(1-z)} \langle k_1 k_2 \rangle} \\ \text{split}_{+}^{q \rightarrow qg}(k_1, -; k_2, +) &= \sqrt{2} \frac{z^{3/2}}{\sqrt{z(1-z)} \langle k_1 k_2 \rangle}, \end{aligned} \quad (42)$$

and with  $\text{split}_{\nu}^{q \rightarrow qg}(k_1, -\nu; k_2, -\nu_2)$  obtained from  $\text{split}_{-\nu}^{q \rightarrow qg}(k_1, \nu; k_2, \nu_2)$  by exchanging  $\langle k_1 k_2 \rangle$  with  $[k_2 k_1]$ .

In the soft quark limit,  $k_1 \rightarrow 0$ , the vertex  $qg^* \rightarrow qg$  in eq. (37) yields at most a square-root singularity, and thus as seen previously it does not yield an infrared divergence. In the soft gluon limit,  $k_2 \rightarrow 0$ , eq. (37) reduces to

$$\begin{aligned} & \lim_{k_2 \rightarrow 0} M^{qg \rightarrow qgg}(-p_a, -\nu; k_1, \nu; k_2, \nu_2; p_{b'}, \nu_{b'}; -p_b, -\nu_b) = 2\hat{s}g^2 \\ & \times \left[ i f^{d_2 c' c} \lambda_{d_1 \bar{a}}^c \text{soft}(k_1; k_2, \nu_2; p_{b'}) - (\lambda^{c'} \lambda^{d_2})_{d_1 \bar{a}} \text{soft}(-p_a; k_2, \nu_2; k_1) \right] \\ & \times \frac{1}{\hat{t}} \left[ i g f^{bb'c'} C_{-\nu_b \nu_{b'}}^{gg}(-p_b, p_{b'}) \right], \end{aligned} \quad (43)$$

with  $\text{soft}(k_1; k_2; p_{b'})$  and  $\text{soft}(-p_a; k_2; k_1)$  obtained from eq. (31) by exchanging  $k_1 \leftrightarrow k_2$ .

In the same fashion as eq. (37), the amplitude for  $\bar{q}g \rightarrow \bar{q}gg$  can be analysed [17].

## 2.3 Virtual NLL corrections to QCD amplitudes

The virtual radiative corrections in NLL approximation can be obtained from the one-loop four-parton amplitudes [27]. As examples, I shall give here the gluon-gluon and the

quark-quark amplitudes. In NLL approximation it suffices to consider the dispersive part, which in dimensional regularization and for the gluon-gluon amplitude is [28, 29]

$$\begin{aligned}
& \text{Disp } M_{1-loop}^{gg \rightarrow gg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\
&= M_{tree}^{gg \rightarrow gg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\
&\times g^2 c_\Gamma \left\{ \left( \frac{\mu^2}{-t} \right)^\epsilon \left[ N_c \left( -\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{s}{-t} - \frac{64}{9} - \frac{\delta_R}{3} + \pi^2 \right) - \frac{\beta_0}{\epsilon} + \frac{10}{9} N_f \right] - \frac{\beta_0}{\epsilon} \right\},
\end{aligned} \tag{44}$$

with  $M_{tree}^{gg \rightarrow gg}$  given in eq. (4),  $N_f$  the number of quark flavors,  $c_\Gamma$  given in eq. (21),  $\beta_0 = (11N_c - 2N_f)/3$ , and

$$\delta_R = \begin{cases} 1 & \text{HV or CDR scheme,} \\ 0 & \text{dimensional reduction scheme,} \end{cases} \tag{45}$$

with (HV) the 't Hooft-Veltman or (CDR) the conventional dimensional regularization schemes [27]. The last term in eq. (44) is the  $\overline{\text{MS}}$  ultraviolet counterterm.

To LL accuracy, using eq. (20), eq. (44) reduces to

$$\text{Disp } M_{1-loop}^{gg \rightarrow gg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) = \alpha(t) \ln \frac{s}{-t} M_{tree}^{gg \rightarrow gg}, \tag{46}$$

in agreement with eq. (18).

The virtual radiative corrections to eq. (9) in NLL approximation are [29]

$$\begin{aligned}
& \text{Disp } M_{1-loop}^{qq \rightarrow qq}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\
&= M_{tree}^{qq \rightarrow qq}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\
&\times g^2 c_\Gamma \left\{ \left( -\frac{\mu^2}{t} \right)^\epsilon \left[ -\frac{N_c^2 - 1}{2N_c} \left( \frac{4}{\epsilon^2} + \frac{6}{\epsilon} \right) + N_c \left( \frac{2}{\epsilon} \ln \frac{s}{-t} + \frac{19}{9} - \frac{2\delta_R}{3} + \pi^2 \right) \right. \right. \\
&\quad \left. \left. + \frac{\beta_0}{\epsilon} - \frac{10}{9} N_f + \frac{1}{N_c} (7 + \delta_R) \right] - \frac{\beta_0}{\epsilon} \right\}.
\end{aligned} \tag{47}$$

To LL accuracy, eq. (47) reduces to

$$\text{Disp } M_{1-loop}^{qq \rightarrow qq} = \alpha(t) \ln \frac{s}{-t} M_{tree}^{qq \rightarrow qq}, \tag{48}$$

which is exactly the same form as eq. (46), due to the universality of the LL contribution.

### 3 Conclusions

We have argued that in order to perform a detailed analysis of dijet production with a rapidity gap we need an  $\mathcal{O}(\alpha_s^4)$  calculation including the relevant collinear enhancements which give structure to the jets. Such a calculation needs not be exact, but must include the full leading power in  $\hat{s}/\hat{t}$  of dijet production in the high-energy limit. The QCD amplitudes must therefore be determined to the corresponding accuracy. We have shown this in detail for the scattering amplitudes necessary to compute the full leading power in  $\hat{s}/\hat{t}$  of dijet production to  $\mathcal{O}(\alpha_s^3)$ .

## A Spinor Algebra in the Multi-Regge kinematics

We consider the scattering of two gluons of momenta  $p_a$  and  $p_b$  into  $n + 2$  gluons of momenta  $p_i$ , where  $i = a', b', 1 \dots n$ . Using light-cone coordinates  $p^\pm = p_0 \pm p_z$ , and complex transverse coordinates  $p_\perp = p_x + ip_y$ , the gluon 4-momenta are,

$$\begin{aligned} p_a &= (p_a^+, 0; 0, 0) , \\ p_b &= (0, p_b^-; 0, 0) , \\ p_i &= (|p_{i\perp}|e^{y_i}, |p_{i\perp}|e^{-y_i}; |p_{i\perp}| \cos \phi_i, |p_{i\perp}| \sin \phi_i) , \end{aligned} \tag{49}$$

where to the left of the semicolon we have the  $+$  and  $-$  components, and to the right the transverse components.  $y$  is the gluon rapidity and  $\phi$  is the azimuthal angle between the vector  $p_\perp$  and an arbitrary vector in the transverse plane. Momentum conservation gives

$$\begin{aligned} 0 &= \sum p_{i\perp} , \\ p_a^+ &= \sum p_i^+ , \\ p_b^- &= \sum p_i^- . \end{aligned} \tag{50}$$

For each massless momentum  $p$  there is a positive and negative helicity spinor,  $|p+\rangle$  and  $|p-\rangle$ , so we can consider two types of spinor products

$$\begin{aligned} \langle pq \rangle &= \langle p- | q+ \rangle \\ [pq] &= \langle p+ | q- \rangle . \end{aligned} \tag{51}$$

Phases are chosen so that  $\langle pq \rangle = -\langle qp \rangle$  and  $[pq] = -[qp]$ . For the momentum under

consideration the spinor products are

$$\begin{aligned}
\langle p_i p_j \rangle &= p_{i\perp} \sqrt{\frac{p_j^+}{p_i^+}} - p_{j\perp} \sqrt{\frac{p_i^+}{p_j^+}}, \\
\langle p_a p_i \rangle &= -\sqrt{\frac{p_a^+}{p_i^+}} p_{i\perp}, \\
\langle p_i p_b \rangle &= -\sqrt{p_i^+ p_b^-}, \\
\langle p_a p_b \rangle &= -\sqrt{p_a^+ p_b^-} = -\sqrt{s_{ab}},
\end{aligned} \tag{52}$$

where we have used the mass-shell condition  $|p_{i\perp}|^2 = p_i^+ p_i^-$ . The other type of spinor product can be obtained from

$$[pq] = \pm \langle qp \rangle^*, \tag{53}$$

where the  $+$  is used if  $p$  and  $q$  are both ingoing or both outgoing, and the  $-$  is used if one is ingoing and the other outgoing.

In the multi-Regge kinematics, the gluons are strongly ordered in rapidity and have comparable transverse momentum:

$$y_{a'} \gg y_1 \gg \dots y_n \gg y_{b'}; \quad |p_{i\perp}| \simeq |p_{\perp}|. \tag{54}$$

Then the momentum conservation (50) in the  $\pm$  directions reduces to

$$\begin{aligned}
p_a^+ &\simeq p_{a'}^+, \\
p_b^- &\simeq p_{b'}^-,
\end{aligned} \tag{55}$$

and the Mandelstam invariants become

$$\begin{aligned}
s_{ab} &= 2p_a \cdot p_b \simeq p_{a'}^+ p_{b'}^- \\
s_{ai} &= -2p_a \cdot p_i \simeq -p_{a'}^+ p_i^- \\
s_{bi} &= -2p_b \cdot p_i \simeq -p_i^+ p_{b'}^- \\
s_{ij} &= 2p_i \cdot p_j \simeq |p_{i\perp}| |p_{j\perp}| e^{y_i - y_j} = p_i^+ p_j^- \quad (y_i \gg y_j),
\end{aligned} \tag{56}$$

where  $i, j = a', b', 1 \dots n$ . In this limit the spinor products (52) become

$$\begin{aligned}
\langle p_a p_b \rangle &\simeq \langle p_{a'} p_b \rangle \simeq -\sqrt{\frac{p_{a'}^+}{p_{b'}^+}} |p_{b'\perp}| \\
\langle p_a p_{b'} \rangle &\simeq \langle p_{a'} p_{b'} \rangle = -\sqrt{\frac{p_{a'}^+}{p_{b'}^+}} p_{b'\perp}
\end{aligned}$$



$$\begin{aligned}
\langle p_a p_{a'} \rangle &\simeq -p_{a'\perp} \\
\langle p_{b'} p_b \rangle &\simeq -|p_{b'\perp}| \\
\langle p_a p_i \rangle &\simeq \langle p_{a'} p_i \rangle = -\sqrt{\frac{p_{a'}^+}{p_i^+}} p_{i\perp} \\
\langle p_i p_b \rangle &\simeq -\sqrt{\frac{p_i^+}{p_{b'}^+}} |p_{b'\perp}| \\
\langle p_i p_{b'} \rangle &\simeq -\sqrt{\frac{p_i^+}{p_{b'}^+}} p_{b'\perp} \\
\langle p_i p_j \rangle &\simeq -\sqrt{\frac{p_i^+}{p_j^+}} p_{j\perp} \quad (y_i \gg y_j) .
\end{aligned} \tag{57}$$

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